**Rice Yield Estimation in India Based on Climate Reanalysis and Remote Sensing for Enhanced Agricultural Early Warning Systems**

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**Abstract**

**1. Introduction**

**1.1. The societal implications of accurate crop yield forecasting in India**

Add section on why early warning systems should include crop yield forecasting

Yield forecasting is the science of predicting agricultural productivity as measured by crop yield – the ratio of the total mass of the harvested product (such as rice) to the area used to cultivate the crop1 – before the harvest takes place, typically a few months in advance.

Pre-harvest prediction of crop yields is important in helping a wide range of stakeholders make better decisions around agricultural planning. For farmers, accurate crop yield forecasts can facilitate decision-making around what to grow and when to grow it2. In addition, near real-time monitoring of crop growth can inform the use of preventive measures such as irrigation and fertilization to boost agricultural productivity where needed3. For governments, yield prediction is relevant to the formulation of policies related to national food security, such as pricing policies for domestic markets, and policy decisions on the import and export of different crops4.

Accurate crop yield forecasting may also enable better design of insurance products that mitigate climate risks and stabilize farmer incomes5. Weather-based crop insurance, for instance, uses a weather index such as total precipitation to determine payments to farmers, meaning that insurance companies do not need to visit farmers to assess damages and arbitrate claims. Rather, if the weather reaches a certain threshold, rapid automatic payments are distributed to farmers, who avoid the need to sell assets to survive due to adverse climate events6.

The need for accurate information on crop yields is particularly important in countries like India, where the agricultural sector provides livelihoods for hundreds of millions of farmers, with 70% of rural households depending on agriculture for their main source of income7.One of India’s major staple crops is rice, which contributes to 30% of calories consumed in India and is a key export commodity for the country8. India cultivates rice on about 45 million hectares of land, with a total production of 178 million metric tonnes in 20209.

**1.2. Overview of approaches and variables that have been used to model crop yields**

Crop yield prediction is a challenging problem in precision agriculture, as final yields depend on a variety of factors such as weather, climate, soil, seed type, and agronomic practices such as irrigation and fertilizer use10.

This complexity is evident from the variety of variables included and methods applied in the growing body of literature on crop yield forecasting. For example, recent examples in literature involving deep learning approaches include corn and soybean yield forecasting in the US based on convolutional neural networks (CNNs) and recurrent neural networks (RNNs)11, soybean yield forecasting in Argentina based on deep transfer learning12, and vineyard grape yield estimations based on CNNs. Recent examples based on machine learning approaches include sugarcane yield prediction using random forests13, prediction of wheat, barley, and canola yields in Western Australia using random forest14, yield forecasting of spring maize in Pakistan based on LASSO regression and support vector machine15, and Jojoba yield prediction in Israel based on gradient boosted regression trees16. Other examples of the machine learning approaches that have been applied to yield prediction have been summarized in a systematic literature review, which also analyzed the variables most frequently included in crop yield prediction studies. Across 50 studies between 2008 and 2019, features used as predictors of yield have included temperature, soil type, rainfall, humidity, pH-value, NDVI, wind speed, and more2.

For studies specific to rice, the staple crop of over half the world’s population17, a number of approaches have been applied to yield forecasting in recent years. Recent examples include rice yield prediction for 81 counties in southern China based on recurrent neural networks18; application of the ecological distance algorithm to model rice yields19; field and county-level rice yield prediction based on synthetic aperture radar (SAR), optical and meteorological data20; random forest yield prediction based on high-resolution imagery collected from unmanned ariel vehicles (UAVs)21; simulation of yields using the Cropping System Model-CERES-RICE22; pixel-scale rice yield prediction in South Korea based on a combination of deep learning and crop models23; rice yield estimation at 500m spatial resolution based on gradient boosted regression and vegetation indices derived from the Moderate Resolution Imaging Spectroradiometer (MODIS)24; and rice paddy yield prediction using sentinel-based optical and SAR data in India based on random forest25.

**1.3. Research contributions**

Experiment-based model that iteratively

Visualization of outputs into a diagnostic dashboard to understand model bias

Detailed reporting of out-of-sample results

Discussion on how the results should be integrated into early warning systems

The present work aims to build on previous efforts in the crop yield prediction literature by building models capable of predicting rice yields at the district level for 362 districts across India. To this end, the present study brings several innovations over the existing literature.

Firstly, an automated machine learning (AutoML) approach is applied in order to test a wide range of models, whereas previous literature has largely focused on yield prediction using a narrow range of algorithms such as random forest or support vector machine2. Secondly, a novel combination of data sources is used to predict Indian rice yields. These include data from ERA5, a climate re-analysis product developed by the European Centre for Medium Range Weather Forecasts (ECMWF), which combines observations with modelled data to provide hourly data on atmospheric, land-surface, and sea-state parameters globally26. Vegetation data was also derived from the MODIS sensors on-board NASA’s TERRA and AQUA satellites. Thirdly, we present an approach capable of forecasting Indian rice yield three months prior to the end of the growing season.

**Open-source computational pipelines to fetch climate data programmatically.** Eded

**Visualization of outputs into a diagnostic dashboard to understand model bias.** XX

**2. Methodology**

**2.1. Study area and overview of methodology**

In this study, climate and remote sensing data will be used as predictors to model rice yields for the kharif season (wet summer monsoon season) from 1999 to 2020 at the district level in India (India consists of 36 states and 684 districts). In India, more than half of the annual rice crop is grown during kharif27, a season which is characterized by high temperature, high humidity, and medium to high rainfall 28. Kharif season rice is typically sowed between the start of June to the end of August and harvested between the end of September to early January, depending on the region 29. During the 2019-2020 season, harvesting of Kharif rice was completed in February 2020.

The methods that will be applied in this study can be summarized as follows. Firstly, 20 years of data on climate, vegetation, and rice yields will be collected from various sources. Next, climate and vegetation data will be pre-processed and aggregated to the same level as historical rice yields prior to development of machine learning models. Lastly, a range of models including Bayesian ridge regression and LightGBM will be trained, evaluated, and interpreted, after which the implications of the modelling results will be discussed with regards to their inclusion within early warning systems.

**2.2. Data Collection**

**Climate reanalysis data.** Daily climate reanalysis data on temperature, potential evaporation, surface pressure, leaf area index, total precipitation, and soil water content was obtained from ERA5 data from the European Center for Medium-Range Weather Forecasts, which provides global estimates of surface and atmospheric parameters since 1950 at a resolution of approximately 30\*30 km30. Climate reanalysis data, which are often freely available, provide temporally and spatially homogenous data31, which makes them suitable for applications such as crop yield prediction in contexts where in-situ weather station measurements are inadequate or incomplete. In addition, weather stations vary in their accuracy and generally record a limited number of variables, such as rainfall, temperature, pressure, and wind speed; variables that are more technically demanding to measure, such as humidity and solar radiation, may be lacking32.

The climate variables used in this study were selected due to their influence on rice yields. An extensive body of research has shown that rice growth is affected by factors such as soil water content33, temperature34, potential evaporation (as a proxy for transpirational demand)35, surface pressure36, and precipitation37.

**Remote sensing data**. Normalized Difference Vegetation Index (NDVI) is a dimensionless index that describes the difference between visible and near-infrared reflectance of vegetation cover, and can be used to estimate the density of green on an area of land38. To determine the density of green on a patch of land, the wavelengths of visible and near-infrared sunlight reflected by the plants are observed. NDVI values range from −1 to +1; higher values of NDVI imply healthy and dense vegetation, whereas lower NDVI values indicate sparser vegetation. NDVI data was obtained from the Moderate Resolution Imaging Spectroradiometer (MODIS) onboard NASA’s Terra and Aqua satellites, due to their wide coverage and temporal resolution. There are several examples in academic literature of using NDVI to investigate the progress of crops, such as for wheat in Argentina39, cereals in Europe40, and rice in Vietnam41. NDVI data was masked using CROPGRIDS, a global, geo-referenced dataset providing information on areas for 173 crops circa the year 2020, at a resolution of 0.05° (~5.55 km at the equator).42

A map of different colors

Description automatically generated

**Figure 1.** Exampleof monthly averaged climate reanalysis data over the India region in a given month for the variables temperature, total precipitation, leaf area index, potential evaporation, surface pressure, and soil water volume.

**Yield data.** District-level rice production and yield data from 1995 to 2021 for 367 districts were obtained from the APY dataset of the Directorate of Economics and Statistics in India’s Ministry of Agriculture and Farmers Welfare43. In this dataset, the year denotes the year in which the crop was harvested. For kharif season rice, the sowing is in the previous calendar year44. Rice yields in India’s top rice-growing regions of Uttar Pradesh, Punjab, Andhra Pradesh, Tamil Nadu, Chhattisgarh, and Haryana have typically ranged from 1 to 5 t/ha between 1995 and 2020, as shown in figure 3.

**Table 1: Data overview.** A range of agronomically-relevant datasets were used as predictors of the target variable (district-level rice yield in India); rice areas masks were used to filter NDVI data by rice-growing area42,43,45,46

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| --- | --- | --- | --- | --- |
| Data type | Parameter | Description | Unit | Source |
| Climate reanalysis | Potential evaporation (pev) | A measure of the extent to which near-surface atmospheric conditions are conducive to the process of evaporation. | m | ECMFW (ERA5) |
| Climate reanalysis | 2m-temperature | The temperature of air at 2m above the surface of land, sea or inland waters. 2m temperature is calculated by interpolating between the lowest model level and the Earth's surface, taking account of the atmospheric conditions. | K | ECMFW (ERA5) |
| Climate reanalysis | Total precipitation | The accumulated liquid and frozen water, comprising rain and snow, that falls to the Earth's surface. It is the sum of large-scale precipitation and convective precipitation. | m | ECMFW (ERA5) |
| Climate reanalysis | Leaf area index, low vegetation | The surface area of one side of all the leaves found over an area of land for vegetation classified as 'low'. 'Low vegetation' consists of crops and mixed farming, irrigated crops, short grass, and more.47 | m2m-2 | ECMFW (ERA5) |
| Climate reanalysis | Total precipitation | The accumulated liquid and frozen water, comprising rain and snow, that falls to the Earth's surface. It is the sum of large-scale precipitation and convective precipitation. | m | ECMFW (ERA5) |
| Climate reanalysis | Volumetric soil water | The volume of water in soil layer 1 (0 - 7cm, the surface is at 0cm) | m3m-3 | ECMFW (ERA5) |
| Remote Sensing | Normalized Difference Vegetation Index (NDVI) | A dimensionless index that describes the difference between visible and near-infrared reflectance of vegetation cover, and can be used to estimate the density of green on an area of land | (-) | NASA EOSDIS (AQUA MODIS) |
| Crop mask | Rice crop mask | A comprehensive global, geo-referenced dataset providing information on areas for 173 crops circa the year 2020, at a resolution of 0.05° (~5.55 km at the equator). | ha | CROPGRIDS |
| Rice production | District-level yield | District-level rice production and yield data from 1995 to 2021 for 367 districts of India. | t/ha | APY |

Data used in this analysis were programmatically downloaded via API and automated Python scripts. ERA5 data was ingested via the CDS API, while NDVI data was ingested via USGS’ AρρEEARS API.

**2.3. Data pre-processing**

Climate data from ERA5 in NetCDF format over a bounded area comprising India was clipped to the Indian country boundary. NDVI data from NASA’s AQUA MODIS satellite in NetCDF format were clipped to the Indian country boundary and masked with a rice cropland layer.

Next, the climate variables and NDVI were aggregated to the district level based on zonal statistics. The vector geometry data for India’s ADM2 (district-level) boundaries which raster pixels were aggregated to were obtained from the Database of Global Administrative Areas (GADM)48. District-level yield data from APY was then merged to the climate and remote sensing data aggregated at the district level to produce a spatially consistent geodataframe. Yield outliers beyond three standard deviations were removed as they were assumed not achievable at the district level in India24.

Feature engineering was conducted to produce monthly averages for the climate and NDVI parameters for every month between May and November, corresponding to the full sowing and growing period for kharif rice24. This process was repeated for all variables to produce a set of 52 features used as input for the modelling.

**2.4. Model development & interpretation**

This study developed and tested the performance of multiple rice yield prediction models based on a variety of machine learning models. These included LightGBM49, an efficient and distributed gradient boosting framework that uses tree-based learning, Bayesian ridge regression50–52, which has been recognized for its ability to deal with hierarchical data structures53, gradient boosting regression54, random forest55, Huber regression56, decision tree regression57, elastic net regression58, AdaBoost59, orthogonal matching pursuit60, and extremely randomized trees61.

The models above were trained on district-level data for 2001 to 2018 (4,606 observations), and validated on out-of-sample test data for 2019 and 2020 (502 observations). The top-performing models were evaluated based on three out-of-sample performance measures including R2, Mean Absolute Error (MAE), and Root Mean Square Error (RMSE). Also reported were Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE), and Root Mean Squared Logarithmic Error (RMSLE). In addition, model results were evaluated based on prediction error plots, residual plots, and spatio-temporal plots of prediction error to evaluate potential model bias (for example, better model performance for certain rice-growing regions of India). Lastly, Shapley Additive exPlanations (SHAP) were used to explore the impact of features on model output62,63.

**2.5. Computation**

Data ingestion, pre-processing, and modelling was conducted in a conda-based python environment with a diverse set of python libraries. Data processing and geospatial operations were carried out using python libraries such as numpy, xarray, pandas, rasterio, rasterstat, and geopandas. Modelling and visualization was conducted using python libraries such as scikit-learn, pycaret, matplotlib, and seaborn.

All code

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